

1) Całkowanie numeryczne: kwadratury:

Newtona-Cotesa: prostokątów, trapezów, parabol; Gaussa: 2-punktowa, 3- punktowa:

a)  $S(f) = (b - a) f(x_0)$ ,

b)  $S(f) = \frac{b-a}{2} [f(a) + f(b)]$ ,

c)  $S(f) = \frac{1}{3} h (f_0 + 4f_1 + f_2)$ ,  $h = \frac{b-a}{2}$ ,

d)  $S(f) = \frac{b-a}{2} \sum_{i=0}^n w_i f\left(\frac{a+b}{2} + \frac{b-a}{2} \xi_i\right)$ ,

gdzie:  $\xi_2 = \left[\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right]$ ,  $\mathbf{w}_2 = [1, 1]$ ,  $\xi_3 = [\sqrt{0.6}, 0, -\sqrt{0.6}]$ ,  $\mathbf{w}_3 = \left[\frac{5}{9}, \frac{8}{9}, \frac{5}{9}\right]$

2) Szereg Taylora:

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + \frac{1}{6}h^3 f'''(x) + \frac{1}{24}h^4 f^{IV}(x) + \dots$$

3) Problem początkowy: metody:

Eulera, polepszona Eulera, Runge-Kutty II rzędu, Runge-Kutty III rzędu, Runge-Kutty IV rzędu dla  $x_{n+1} = x_n + h$

a)  $y_{n+1} = y_n + k_1$ ,  $k_1 = h \cdot f(x_n, y_n)$ ,

b)  $y_{n+1} = y_n + k_2$ ,

$$k_1 = h \cdot f(x_n, y_n), \quad k_2 = h \cdot f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right),$$

c)  $y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$ ,

$$k_1 = h \cdot f(x_n, y_n), \quad k_2 = h \cdot f(x_n + h, y_n + k_1),$$

d)  $y_{n+1} = y_n + \frac{1}{4}(k_1 + 3k_3)$ ,

$$k_1 = h \cdot f(x_n, y_n), \quad k_2 = h \cdot f\left(x_n + \frac{1}{3}h, y_n + \frac{1}{3}k_1\right), \quad k_3 = h \cdot f\left(x_n + \frac{2}{3}h, y_n + \frac{2}{3}k_2\right),$$

e)  $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ ,

$$k_1 = h \cdot f(x_n, y_n), \quad k_2 = h \cdot f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right),$$

$$k_3 = h \cdot f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right), \quad k_4 = h \cdot f(x_n + h, y_n + k_3).$$

4) Problem brzegowy:

$$f' = \frac{1}{2h} (-1f_{i-1} + 1f_{i+1}),$$

$$f'' = \frac{1}{h^2} (1f_{i-1} - 2f_i + 1f_{i+1}),$$

$$f''' = \frac{1}{2h^3} (-1f_{i-2} + 2f_{i-1} - 2f_{i+1} + 1f_{i+2}),$$

$$f^{IV} = \frac{1}{h^4} (1f_{i-2} - 4f_{i-1} + 6f_i - 4f_{i+1} + 1f_{i+2})$$

5) Krok potęgowy i iloraz Rayleigha

$$\mathbf{x}^{(k+1)} = \mathbf{A} \mathbf{u}^{(k)},$$

$$\lambda^{(k+1)} = (\mathbf{u}^{(k)})^T \mathbf{A} \mathbf{u}^{(k)} = (\mathbf{u}^{(k)})^T \mathbf{x}^{(k+1)}$$