Higher order meshless schemes applied to the finite element method in elliptic problems

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This research concerns FEM and meshless methods coupling on two levels of numerical analysis, namely

1. *discretization level*: division of a problem domain into disjoint subdomains and application of a different approximation and discretization model in each subdomain, independent from each other and

2. *approximation level*: application of higher order approximation schemes, typical for meshless FDM, for FE framework.

Seminar will be devoted to selected aspects of (2). Finite elements with standard or hierarchical shape functions are coupled with higher order meshless schemes, based upon the correction terms of a simple difference operator. Those terms consist of higher order derivatives, which are evaluated by means of the appropriate formulas composition as well as a numerical solution, which corresponds to the primary interpolation order, assigned to element shape functions. Correction terms modify the right-hand sides of algebraic FE equations only, yielding an iterative procedure. Therefore, neither re-generation of the stiffness matrix nor introduction of any additional nodes and/or degrees of freedom is required. Such improved FE-MFD solution approach allows for the optimal application of advantages of both methods, for instance, a high accuracy of the nodal FE solution and a derivatives' super-convergence phenomenon at arbitrary domain points, typical for the meshless FDM. Existing and proposed higher order techniques, applied in the FEM, are compared with each other in terms of the solution accuracy, algorithm efficiency and computational complexity.

In order to examine the considered algorithms, numerical results of several two-dimensional benchmark elliptic problems are presented. Both the accuracy of a solution and the solution's derivatives as well as their convergence rates, evaluated on irregular and structured meshes as well as arbitrarily irregular adaptive clouds of nodes, are taken into account.

Moreover, short presentation of a post-processing stage is planned. Especially, effective techniques of mesh generation, based upon *gmsh* software, will be discussed, including a-priori domain partition into subdomains. It is followed by the application of higher order schemes, regardless of the numerical model applied in each subdomain.